Mobility Erosion: High speed motion safety for mobile robots operating in off-road terrain

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Abstract— **This paper addresses the problem of ensuring mobile robot motion safety when reacting to soft and hard hazards in a static environment. The work is aimed at off-road navigation for mobile ground robots where soft hazards are posed by varying terrain conditions (e.g. deformable soil, slopes, vegetation). Soft hazards pose operating constraints (i.e. speed limits) to the mobile robot that need to be satisfied to ensure motion safety. This paper presents a new morphological erosion operator that generalizes binary obstacle growing to mobility space (the space of speed limits) to deal with both hard and soft hazards seamlessly. This ensures that topological constraints due to vehicle size as well as momentum are taken into account, and leads to a straight-forward approach to generalize the concept of 'regions of inevitable collision' for soft hazards.**

I. Introduction

A. Overview of the problem

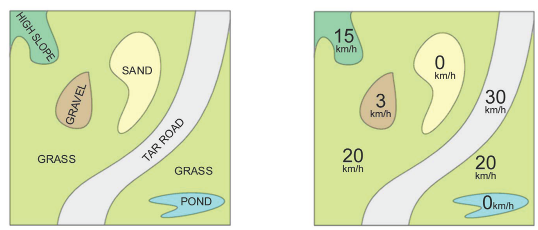
Mobile robot motion safety with respect to hard (i.e. inpenetrable) hazards such as static and dynamic obstacles is well studied in the literature. There are many algorithms that address reactive decision making for hard hazards like obstacles. Methods such as artificial potential fields [12], Vector field histogram (VFH) approach [1] and the Dynamic window approach (DWA) [5] all address the case of hard hazards alone. Extending these reactive algorithms to soft hazards while retaining their safety guarantees is of value for off-road ground vehicle navigation.

Soft hazards are terrain regions that are not untraversable like obstacles, but they require the vehicle to adapt its behavior to safely negotiate them. For example, a mobile robot may need to slow down while traveling downhill to eliminate the risk of excessive slippage and loss of control. A formal definition of soft hazards is given below:

Definition 1: Soft hazards are environment conditions that impose additional differential constraints to the vehicle beyond the non-holonomic constraints arising from its dynamics.

Soft hazards are traditionally addressed by scoring preferences for different state transitions in the form of a cost function. The cost function forms an input into a planning algorithm that reasons on the preferences for state transitions. However, the cost function is only a comparative assessment, and is only useful in path planning. It does not provide any absolute quantification so as to guarantee motion safety, which is required for reactive decision making. In this paper, we provide a unified interpretation of motion safety for hard and soft hazards by representing the environment in terms of state space constraints (position, heading and velocity).

In this representation, hazards are represented as different degrees of speed (or kinetic energy) limits as a function of position and heading. A hard hazard has associated with it a zero speed limit. A soft hazard has some positive value below the maximum vehicle speed limit. These constraints provide a metric for absolute quantitative assessment of terrain conditions. A physical interpretation of such a mobility representation is shown in Figure 1, where different terrain conditions are represented in terms of speed limits, to create mobility maps.



(a) Sample terrain classification (b) Mobility map

Fig. 1. Mobility maps used to represent hard and soft hazards in a unified representation

Given these constraints, 'safe' speed and orientation setpoints are determined and provided to low-level steering and speed controllers. This enables a reactive controller to continuously adapt the vehicle's operating speed and heading by reacting to terrain conditions. Existing feed-forward reactive algorithms for hard hazards (e.g. VFH [1]) can then be generalized to soft hazards by performing multiple closed-loop simulations for a fixed sensing horizon given a low-level controller and a vehicle motion model. In addition to satisfying constraints posed by the environment, forward simulation with a motion model ensures that kinodynamic constraints are satisfied by construction.

This work is aimed at unmanned ground vehicles (wheeled or tracked vehicles) operating in an off-road environment with varying terrain conditions (e.g. deformable soil, 2D slopes, vegetation). Moving hazards are not considered, however the presented work can in principle be fused with existing algorithms [4] that address moving obstacles.

B. Contributions and paper outline

There are two main contributions of this paper i) A new conceptual development that provides a unified interpretation

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of the *regions of inevitable collisions* [13] for hard and soft hazards and ii) A new morphological operator known as *mobility erosion* that serves as a practical tool to enforce the former conceptual contribution.

Mobility erosion is significant because a) It can ensure motion safety by taking into account vehicle momentum, in addition to its size and shape c) It is a local operation that can be used within reactive algorithms that operate with a limited field of view of the environment. These properties enable generalization of reactive controllers [1], [5] to soft hazards while ensuring motion safety.

This paper is structured as follows. Section II introduces the concepts necessary to establish motion safety for hard and soft hazards. Section III discusses some relevant work. Sections IV-A and IV-B develop the new morphological operator that generalizes obstacle growing to mobility space. Section V presents some results that demonstrate the utility of mobility erosion for reactive decision making. Concluding remarks are presented in Section VI.

II. Guaranteed motion safety

A. Motion safety for hard hazards

In the context of collision avoidance, the notion of motion safety implies choosing controls that guarantee a collision free transition between vehicle states. In order to determine if a configuration is collision free, one needs to consider the size and shape of the vehicle and the obstacle. In addition to size and shape, vehicle speed also influences a collision free transition as it determines the minimum stopping distance. The role of speed in motion safety can be best explained by illustrating the concept of '*regions of inevitable collision (RIC)*' first introduced by [13] or alternatively referred to as inevitable collision states (ICS) by [6]. States that lead to a collision regardless of the choice of control actions are referred to as the regions of inevitable collision (or RIC). Figure 2 shows a 1D illustration with position on x-axis and speed on y-axis, and illustrates the effect of speed on RIC. RIC grows quadratically with respect to speed, as it takes longer for the vehicle to stop.

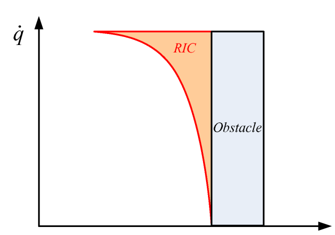


Fig. 2. Region of inevitable collision (RIC) grows quadratically with speed. X-axis is position and Y-axis is speed.

Validation for motion safety with hard hazards is reduced to determining states that result in a collision free transition regardless of the choice of control action. This notion of safety can be quantified as posing constraints in configuration space (position and heading).

B. Motion safety for soft hazards

In the context of soft hazards, vehicle motion safety is here defined as follows:

Definition 2: Vehicle motion among soft hazards is considered to be safe when constraints in *proprioceptive* space are satisfied. Proprioceptive constraints are here defined as limits on longitudinal/lateral slip, traction coefficients, lateral acceleration, vibration, etc.

In order to adjudge soft hazard safety for any given vehicle configuration, the fixed constraints in proprioception space need to be transformed into constraints on vehicle state space (position, heading, speed, turn rate and acceleration). The result of such a process is to explicitly determine the set of feasible state transitions that can be safely chosen without violating proprioceptive constraints.

The transformation from proprioceptive constraints to state space constraints can be achieved in two ways i) via exhaustive forward simulation ii) via an inverse dynamics model that maps proprioceptive space to state space. Both simulation and learning an inverse model involve solving a feasibility problem (an optimization with zero objective and non-zero constraints) for every combination of speed, heading and terrain condition. A stochastic learning based approach to this problem was addressed in previous work [9], [10]. In this paper, it is assumed that an instantaneous mobility function that associates each terrain condition with a speed or kinetic energy limit is given *a priori*. In the case of hard hazards, the mobility function would be a discrete function (0 for hazards and max vehicle speed limit otherwise). The complexity of the mobility function seamlessly generalizes motion safety from hard to soft hazards.

Given independent state space constraints for each environment state (determined from the feasibility problem), the topological influence of neighboring states can be taken into account by considering a *soft interpretation of the RIC*. This topological consistency is required to guarantee motion safety. This paper presents a new gray-scale erosion operation on the mobility space (space of speed limits) that addresses the above. It provides a hard bound on speed that ensures safety for both hard and soft hazards. Instead of growing the size of hazards (position constraints), the mobility (speed constraints) of soft hazards are eroded/reduced as a function of proximity to lower mobility states. The region of topological influence is quadratic with respect to speed.

III. Related methods in robotics

Planning algorithms and reactive controllers often consider the vehicle to be a point mass in the configuration space. In order to account for the shape and size of the vehicle in the physical space, traditional path planning uses the *Minkowski sum* operator ($\oplus_m$) [14] to grow binary obstacles. These *enlarged* obstacles enable the path planning algorithm to ignore vehicle size and treat it as a point mass. However, the Minkowski sum is only applicable for hard hazards. To address soft hazards, unmanned ground vehicle systems implement a morphological operator from image processing literature [7] known as *gray-scale dilation* that generalizes binary obstacle growing to continuous cost spaces [3]. Both Minkowski sum and dilation only compensate for size and do not take vehicle momentum and the subsequent notion of RIC into account. There has been no prior effort to determine RIC in the continuous space of hard and soft hazards which is addressed in this paper.

Existing literature focuses on the problem of determining RIC for the scenario of moving obstacles [2], [4]. Most of these techniques resort to explicit forward simulation given a fixed look-ahead time to determine if there is a collision or not. There is also related work in explicitly defining obstacles in velocity space known as *Velocity Obstacles (VO)* [4] that have to been applied to the problem of reactive decision making to avoid moving obstacles. VO can be thought of as a restricted case of RIC where only future trajectories with constant velocity are considered. [5] and [15] propose a similar concept to deal with hazard avoidance, but instead of calculating the RIC explicitly they aim to determine the admissible space of trajectories/velocities (RIC^C). The solution presented in this section to deal with RIC is similar in spirit to [5], [15]. In the following section, the mobility erosion operator is developed. First, a dual of gray-scale dilation known as gray-scale erosion is formally introduced followed by a generalisation to address RIC.

IV. Mobility Erosion

A. Gray-scale erosion

Similar to a gray-scale dilation on cost spaces, *gray-scale erosion* achieves the same objective in utility (i.e. negative cost) space. From an image processing context, gray-scale erosion enlarges the dark areas in the image and subdues the lighter areas (assuming darker intensities are of lower values). Dilation and erosion are dual set theoretic operators defined on two sets: one of those sets represents the topology or the image under consideration, and the other one is called the structuring element (SE) which is used to process the given image.

Consider a set of spatial states $s \in S$ in a topological arrangement that represents a given environment where the vehicle has to operate (i.e. task space). The states represent all possible vehicle configurations (position and heading) in the given environment. Each position and heading is associated with certain exteroceptive properties (e) (e.g. color and slope) derived from sensor data.

Given a mobility function over these exteroceptive properties ($I = f(e)$), an initial estimate of mobility (i.e. speed or kinetic energy limit) (I) is associated with each of the configurations in the topology ($I(s) = f(e(s))$).

These initial estimates of mobility are over-confident as they assume that the states in the topology are independent (e.g. a high speed area is not affected by its proximity to a low speed area). Gray-scale erosion (see Equation (1)) is used to enforce topological constraints by shrinking (eroding away) high mobility areas due to their proximity to low mobility areas. The extent of erosion is specified by the size of the structuring element (SE) and the mobility values in the neighborhood. SE is a set of offsets to neighboring states that account for the shape and size of the vehicle. By eroding with such a SE, low speed areas are expanded to reflect the increase in caution due to vehicle shape and size.

$$E(s) = \min_{s' \in SE} \{I(s + s')\} \tag{1}$$

In the context of a 2D grid topology, I is the input image consisting of instantaneous mobility values and E is the output image after erosion. SE is a binary matrix referring to the offsets (s') of the neighbors (from the current pixel s) that need to considered when eroding.

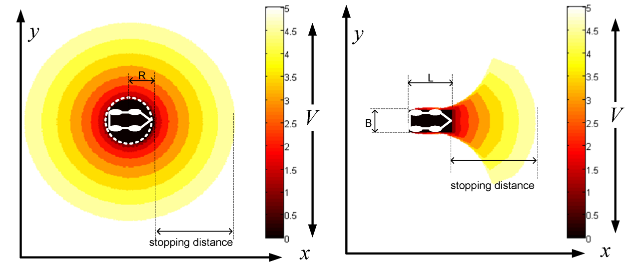
B. Gray-scale erosion with adaptive structuring elements

A generalization of RIC to the continuum of soft and hard hazards can be achieved by using an adaptive SE. The size and shape of the SE is made variable as a function of vehicle size plus worst-case stopping distance. More formally, the structuring element represents the minimal *reachability* space [2] of the vehicle for a look-ahead given by the worst case stopping distance.

The worst-case stopping distance changes according to different terrain conditions. Given an initial estimate of optimistic speed limit defined for that terrain condition (instantaneous mobility value) the worst-case stopping distance can be determined. For a point mass with initial speed limit v (or kinetic energy limit), assuming double integrator dynamics the worst-case stopping distance ($d(v)$) is given as $\frac{v^2}{2a_{\max}}$, where $a_{\max}$ is the maximum possible deceleration that can be achieved in the given terrain. Reachability is also affected by system latency (δ) which adds an increment to the stopping distance ($v \times \delta$). In addition to reachability, position uncertainty can be taken into account in the structuring element by dilating it with the 2-sigma uncertainty ellipse.

Vehicle reachability with and without steering considerations is shown in Figure 3. Each grid value specifies the initial velocity required to be able to reach that point under a braking maneuver. This reachability map was generated by collating forward simulations of braking maneuvers for a discrete set of starting conditions (set of initial velocities and fixed steering angles). The level sets of the reachability map provide the binary neighborhood required to create a SE for a given initial condition. Alternatively an analytical function can be used when appropriate. This process can be performed offline and the SE set for various initial conditions can be stored in memory. Two sample structuring elements are illustrated in Figure 3. The isotropic SE set ignores vehicle orientation to create a conservative estimates of vehicle reachability. The anisotropic SE set shown is for a steered vehicle and it only considers a fixed steering range ($\pm 30°$) for a given position and orientation of the vehicle to generate less conservative reachability estimates.

The mobility erosion operation is given in Equation (2) where the SE is adapted according to the current-state mobility values derived from the mobility function ($I()$). Mobility erosion searches for the optimal speed limit below the prescribed maximum vehicle speed limit($m \in [0, I(s)]$) that maximizes the *minimum mobility in the neighborhood*



(a) Isotropic Reachability (b) Anisotropic Reachability

Fig. 3. A set of structuring elements for isotropic and anisotropic mobility erosion. The above image shows vehicle reachability under braking maneuvers (assuming a fixed worst-case decceleration). The colors correspond to the minimum initial velocity required to reach any given state under worst case braking maneuvers (white -$5m/s$ and black - $0m/s$). Each level set in the above images is a unique SE and corresponds to a mobility (velocity or kinetic energy) state. The isotropic SE set ignores vehicle orientation to create conservative estimates of vehicle reachability. The anisotropic SE set is shown for a steered vehicle and it only considers a fixed steering range ($\pm 30°$) to generate less conservative reachability estimates.

This leads to a *maxmin* formulation as shown below in Equation (2).

$$E(s) = \max_{m \in [0, I(s)]} \left\{ \min_{s' \in SE(m)} \{m, I(s + s')\} \right\} \qquad (2)$$

The end result of applying Equation (2) with a rectangular structuring element (ignoring steering capabilities) is shown in Figure 4 on a 2D topology with the vehicle heading towards a wall. The topology has a linear mobility gradient over the y-axis and it represents a scenario with increasingly dense vegetation (to represent a gradual increase in the traversability of soft hazards). Figure 4(c) shows the difference between the initial mobility map and the eroded mobility map. The quadratic increase in RIC illustrated in the 1D scenario of Figure 2 is now translated into a quadratic decrease in speed limits for the 2D continuous mobility case.

Figure 5(b) shows the result of erosion with an adaptive isotropic structuring element in a open field of obstacles (Figure 5(a)). The radius of the circular structuring element was determined by the vehicle size plus the stopping distance. The isotropic circular structuring element is the simplest to implement, however it leads to conservative speed limits. Less conservative speed limits can be achieved by considering a orientation sensitive structuring element at the cost of increased computational complexity. This results in anisotropic erosion, which is performed in the 3D space of position (x,y) and heading[1]. Figures 5(c)-5(d) illustrate anisotropic erosion for a anisotropic structuring element that is rotated with respect the value of the orientation. Figures 5(c)-5(d) are slices at a subset of orientations from the 3D space of position (x,y) and heading. Similarly, Equation (2) also applies for the case of soft hazards; Figure 6(b) shows the end result of isotropic mobility erosion in a topology with discrete obstacles and varying mobility conditions.

Mobility erosion given by Equation 2 is a local operation and does not require complete knowledge of the environment a priori. It can be performed with limited local views from immediate sensing and also can also be dynamically updated when the some mobility values of the environment change over time. The later details are not discussed in this paper as it is beyond the scope. Mobility erosion involves performing a single convolution-like operation whose computational complexity is $O(NM)$ where N is the number of pixels in the scene and M is the no. of pixels in the largest SE. The computational complexity is mostly linear with the environment size in consideration which is usually the sensor field of view.

V. Results

In previous work [10], [11], eroded mobility maps were used as a cost functions into path planning and were also used to generate terrain adaptive paths in off-road environments with soft hazards. In this paper, the focus is on demonstrating the utility of mobility erosion for reactive decision making to illustrate its significance for motion safety. Simulation results are presented that show improvements to hard hazard avoidance at high speeds. Figures 7(a) and 7(b) show 500 simulations of reactive hazard avoidance with randomly selected start and goal locations with an assumed maximum vehicle speed limit of $5m/s$. Paths colored red resulted in a collision. Magenta paths could not reach the goal and were safely terminated. Blue paths are collision free and were successful. The hazard avoidance was performed using the vector field histogram (VFH) algorithm [1], with and without erosion given a fixed lookahead distance of 10m. The set of 500 simulations were repeated for increasing values of the maximum vehicle speed limit as summarized in Table I.

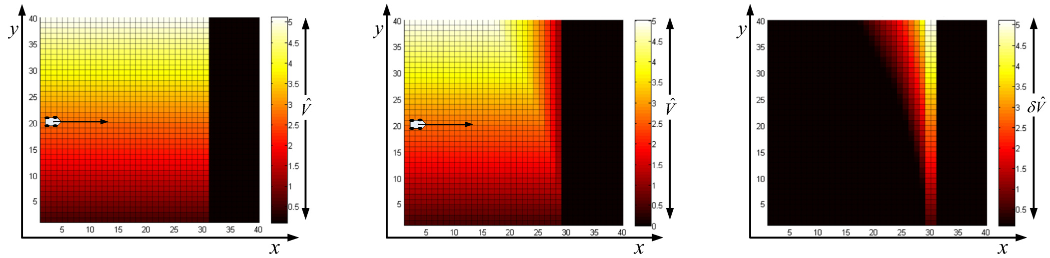
The simulations are were performed through a closed loop system of low-level steering and speed controllers and a vehicle motion model. An empirically tuned steering dynamics model presented in [8] for a four wheeled Ackermann steered all-terrain vehicle is used for simulations. The model parameterizes the steering dynamics as a second order system[2] with an actuator time delay of $0.2s$. Similarly, a proportional speed controller (gain $= 10$) and double integrator dynamics were used to simulate velocity regulation. Finally, a kinematic motion model [3] for a steered vehicle was used to couple the velocity and steering dynamics.

The erosion values were incorporated into the VFH algorithm in two stages. First, the eroded values are used to create a mobility envelope which is used to choose the orientation with maximum projection to goal as a setpoint. Second, feedforward simulations of the closed loop system discussed previously are directed towards a set of goals for a fixed look-ahead so as to create a set of intended directions to choose from as in the VFH algorithm[3]. In contrast, the

[1]In this paper, the anisotropic speed limits are represented as a function of position and orientation only. Higher resolution can be obtained at the expense of increased computational complexity by making the speed limits a function of position, orientation and steering angle (or path curvature).

[2]Transfer function - $\left(1/258.7s^2 + 6.5789s + 1\right)$

[3]A minimum time estimate is used to score the simulations.

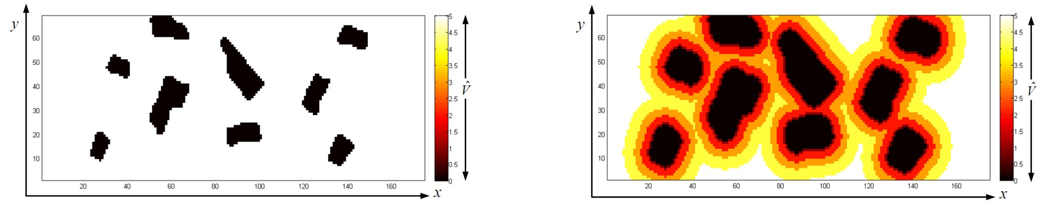


(a) Input: Instantaneous mobility map (units: x/y-m; color-m/s)

(b) Output: Eroded mobility map using Equation (2) (units: x/y-m; color-m/s)

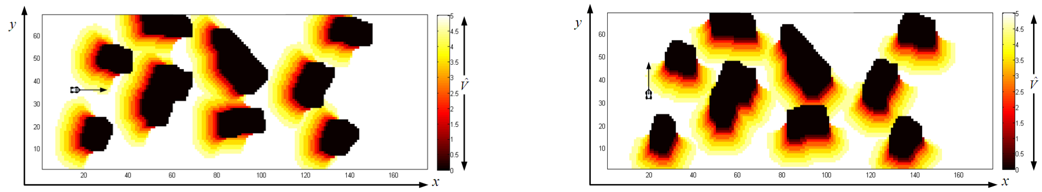
(c) Difference: Instantaneous Mobility − Eroded Mobility

Fig. 4. Mobility erosion on a topology with increasingly dense vegetation (linear gradient) over y-axis leading towards a wall (hard hazard). The vehicle is assumed to oriented and moving towards the wall (as shown in 4(a) and 4(b)). X and Y axes indicate 2D position and the Color axis indicates speed limits (for subfigures a and b) or the change in speed limits (for subfigure c). (color scale: white = $5m/s$ and black = $0m/s$).



(a) Instantaneous Mobility (units: m/s)

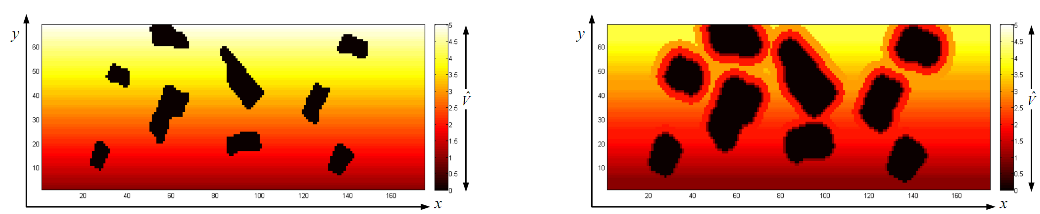
(b) Isotropic erosion with a circular structuring element (units: m/s)



(c) Erosion in $\{x, y, \theta\} : \theta = 0°$

(d) Erosion in $\{x, y, \theta\} : \theta = 90°$

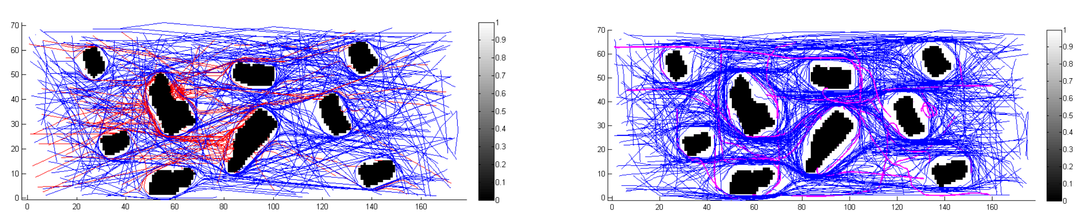
Fig. 5. 5(b) shows the result of isotropic mobility erosion on a topology with hard hazards only (shown in 5(a) - 70m×170m area). Sub-figures 5(c)-5(d) show the result of mobility erosion in $\{x, y, theta\}$ using a anisotropic structuring element on a subset of orientations (the assumed vehicle heading is indicated by an arrow). (Units: x/y-m; Color scale: white = $5m/s$ and black = $0m/s$)



(a) Instantaneous mobility

(b) Eroded mobility using a circular structuring element

Fig. 6. Isotropic mobility erosion on an topology with hard and soft hazards using a circular structuring element. The topology of soft hazards represents a scenario with increasingly dense vegetation (linear gradient) over y-axis and was chosen for illustration purposes.



(a) without erosion at 5m/s with a 10m lookahead

(b) with erosion at 5m/s with a 10m lookahead

Fig. 7. Reactive hazard avoidance for 500 simulations (randomly selected start and goal locations) using Vector Field Histogram with and without erosion. Red paths resulted in a collision. Magenta paths could not reach the goal and were safely terminated. Blue paths are collision free and were successful.

standard VFH algorithm discretizes its local lookahead area in sectors and chooses the closest obstacle free sector to goal. In addition to a desired orientation, the erosion values also provide speed limits that are used to regulate the velocity. Safety is ensured by regulating operating speed to remain under a given speed limit. The desired orientation aims to steer the vehicle away from hard hazards, if a safe orientation is not found due to the suboptimality of VFH the vehicle simply comes to a halt.

Table I shows improvements in reactive hazard avoidance with and without erosion at a set of maximum possible speeds (column one). The table shows significant improvement in the performance of reactive decisions, improving from a 72% success rate and 28% collision rate without erosion to a 98% success rate and 0% collision rate with erosion. 2% of the simulations with erosion could not find a solution to goal (since reactive controllers do not guarantee a solution) but the vehicle was safely halted before any impending collision by regulating velocity to zero (fourth column). The improvement in safety comes from the rich information provided by erosion to regulate velocity to stay below the prescribed speed limits. By lowering operating speed, the RIC is effectively reduced, assuring that enough room is available to avoid RIC within the limited lookahead horizon. Without erosion, the vehicle would change speed or direction abruptly which can be problematic at high speeds. 8% of the random start or goal locations (out of 3000) were pruned out as they were inside the obstacles (second column).

TABLE I

REACTIVE HAZARD AVOIDANCE WITH AND WITHOUT EROSION FOR 500 SIMULATIONS (RANDOMLY SELECTED START AND GOAL LOCATIONS)

V_{max} (m/s)	Invalid start/goal	Collisions (w/o erosion)	Collisions (w/ erosion)	No solutions (w/ erosion)
5	36	83	0	8
10	36	86	0	6
15	36	105	0	8
20	36	133	0	5
25	37	174	0	5
30	37	191	0	5

VI. CONCLUSION

This paper presented an approach to determine operating vehicle constraints (i.e. speed limits as a function of position and heading) for environments with hard and soft hazards. Assuming perfect localization and sensing, motion safety is guaranteed as long as the given operating vehicle constraints are satisfied. In addition, the chosen SE function must generate conservative reachability estimates to guarantee safety.

The other significant contribution of this paper is the mobility erosion operator which is significant for three reasons a) The adaptive structuring element accounts for vehicle size, reachability due to momentum, system latency and position uncertainty. b) It is a local operation that can be performed efficiently and avoids closed loops in calculations. c) In addition to a better choice of steering, the mobility maps also provide information to regulate vehicle speed. However, the end result of erosion is subject to availability of an instantaneous mobility function. Determination of this mobility function was addressed by [10] for the case of 2D off-road slopes.

Some future directions to this work are as follows. Moving obstacles represented using velocity obstacles can be incorporated into the eroded mobility maps as they are both defined in the space of velocity constraints. Localization and sensing uncertainty can be taken into account to generate a graceful degradation of safety guarantees (safe $x\%$ of the time) with increasing uncertainty. Since mobility erosion is a local max-min operation it is not limited to grid representations, it can also be performed in a randomized sampling architecture.

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