A Margin-Based Approach to Threat Assessment for Autonomous Highway Navigation

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Abstract— **In this paper we present a new approach to the threat assessment problem for semi-autonomous and fully autonomous vehicles, based on the estimation of the control freedom afforded to a vehicle. Given sensor information available about the surrounding environment, an algorithm is described for identifying fields of safe travel through which the vehicle can safely navigate. Within each candidate field, we then characterize the level of threat, to influence autonomous navigation or driver support inputs. To characterize threat, the fields of safe travel are associated with sets of feasible trajectories generated from a lattice sampled in the vehicle's input space. A planner then computes a metric associated with available control freedom from these sampled trajectories. This method potentially allows a semi-autonomous control system to honor safe driver inputs while ensuring safe and robust navigation properties. It could also serve as an input to an autonomous decision-making layer.**

I. INTRODUCTION

Recent traffic safety reports from the National Highway Traffic and Safety Administration show that in 2012 alone, over 33,500 people were killed and 2.4 million injured in motor vehicle accidents in the United States [1]. The presence of passive and active safety systems has contributed to a decline in these numbers from previous years. However, even if active safety technologies have begun to play a major role in collision mitigation [10], the need for improved threat assessment methods remains significant. Recent developments in onboard sensing, lane detection and obstacle recognition have facilitated these active safety systems [4]. However, most of these technologies have limited ability to assess the future threat posed to the vehicle: many are reactive in nature, and cannot apply preventive actions based on estimates of potential future actions. Here, a predictive, margin-based threat assessment approach is described that combines sensory information related to the surroundings and information about the host vehicle's dynamic to assess threat associated with future actions. This threat estimate could then be used to plan the desirable navigation option (in an autonomous system) or correct the anticipated driver's trajectory when the threat exceeds a specified threshold (in a semi-autonomous, active safety system).

A. Motivation

This proposed threat assessment method relies on the observation that human drivers tend to operate vehicles within fields of safe travel, rather than along a specific path [6]. Based on this, we propose a framework that relies on identification and analysis of candidate fields, each of which can be interpreted to contain a path homotopy (see figure 1).

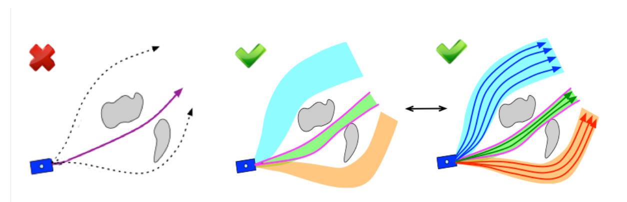

Fig. 1. From path planning to corridor navigation and sets of trajectories

Threat to a vehicle is assumed to be proportional to the freedom of control afforded by a particular field. This implies that (for instance) drivers tend to prefer to navigate through regions that are wide and exhibit low curvature. To characterize threat in each candidate field, a state lattice is constructed via sampling in the vehicle input space. This sampling method enables identification of sets of feasible trajectories associated with each field, from which the metrics related to control freedom can be derived. Lattice algorithms have been extensively studied and are highly flexible tools for motion planning that can adapt to complex environments. Lattice structures can be adapted to parallel computing methods, making them computationally efficient and implementable in real-time at high resolutions [11].

B. Related Work

In the context of autonomous control, typical path planning tools include rapidly exploring random trees [9], potential fields [13], and lattice-based approaches [15],[12],[14] and [7]. These approaches have proven effective in autonomous navigation, however their reliance on specific reference paths (rather than spatial fields) can potentially lead to non-robust performance in the presence of various forms of uncertainty that can cause deviation from the planned path. Also, for semi-autonomous applications, forced adherence to a specific path can feel non-intuitive or over-restrictive to a human operator.

More recently, optimal-control-based methods in a field-based framework [3], [8] propose to tackle the navigation problem by dividing the space into homotopy classes and generating optimal trajectories within a particular class by minimizing a chosen objective function. A drawback to [3] and [2] is their reliance on an ad hoc metric for evaluating the desirability of a candidate field. Also, the threat assessment metric is computed from metrics associated with an optimal path within a particular field, and can therefore be sensitive to uncertainty.

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Other approaches to estimating the future threat posed to a vehicle have been based on reachability analysis, which aims to assess whether there exists a feasible path through an environment to reach a desired goal, considering a driver model [5]. This approach provides a manner for determining the time of intervention, but provides only a binary assessment of reachability, and is therefore limited in its ability to compare the desirability (i.e. 'goodness') of various candidate navigation choices.

The margin-based approach presented in this paper builds on the field-based framework introduced in earlier work by the authors [2] and relies on an input space lattice to assess the threat posed by each candidate field. The proposed threat metric captures both geometric properties of the candidate fields and dynamic properties of the vehicle.

C. Paper outline and technical approach

The threat assessment problem is addressed as in two stages. The first stage identifies all possible fields of safe travel in the local vehicle surroundings. This process is described in section II. The second stage (Section III) characterizes the desirability of each field in order to detect potential hazardous navigation decisions. This second stage is, strictly speaking, the threat assessment problem, which is addressed with a margin-based method that relies on analysis of a state lattice. Finally, in Section IV, these methods are demonstrated in simulated highways scenarios involving multiple tasks such as vehicle overtaking, lane merging, or emergency braking.

II. FRAMEWORK - IDENTIFICATION OF HOMOTOPY CANDIDATES

The threat assessment method presented in this paper relies on identification of fields of safe travel, a choice motivated by the observation that human drivers tend to operate vehicles within spatial fields rather than along a specific path. We will loosely refer to these fields as homotopies. (Strictly speaking, a homotopy of paths is a set of paths derived from continuously deforming a reference path while maintaining fixed path endpoints.)

A. Creation of the decision tree

Consider a multi-lane road environment containing obstacles (i.e. vehicles or other objects) and multiple fields of safe travel toward a goal region. Navigation decisions can be enumerated based on the observation that each time the vehicle encounters an obstacle it faces a choice between remaining in its current lane (by slowing) or moving to an adjacent lane. This illustrated in Fig.2.

We assume the presence of multiple target zones that the driver may wish to reach. These targets are determined as follows. First, target zones are identified immediately behind each vehicle in the environment. This is to accommodate a driver's potential desire to remain behind (i.e. without passing) a particular vehicle. Second, a multi-lane target zone is identified at a user-defined distance from the vehicle (i.e. at the limit of the range of local sensors). This is to accommodate a driver's potential desire to travel in an open lane. The length of each target can be chosen arbitrarily, while the width is constrained by the lesser of the lane width or the free (i.e. non-obstacle) space in the lane.

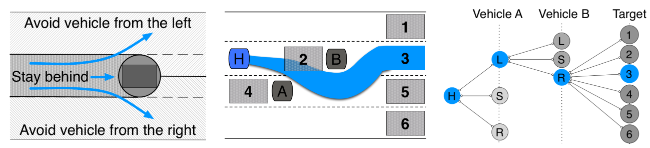

Fig. 2. From left to right: driving strategies, definition of the target regions and construction of the decision tree

A tree structure can be constructed by enumerating all the possible navigation decisions associated with each obstacle, with each branch of the tree terminating (not necessarily uniquely) in a target zone. A homotopy can then be identified as field constructed from connected regions associated with each branch of the decision tree. For each homotopy candidate h_i, $\delta_{i,j}$ is the decision taken at the j^{th} vehicle and τ_i the chosen target zone. By construction, $\tau_i \in \{1,...,N_T\}$ and $\delta_{i,j} \in \{left, behind, right\}$. Each homotopy candidate can be written in the form of a sequence $h_i = (\delta_{i,1},...,\delta_{i,N_V},\tau_i)$ that represents the navigation decision at each level of the decision tree. This method for homotopy candidate identification is flexible as it allows various behaviors of the lead vehicles (i.e. to accelerate, slow down or change lane).

B. Heuristics to prune the number of homotopy candidates

The number N_H of homotopy candidates depends on the number of vehicles N_V and number of target zones N_T, $N_H = 3^{N_V} \times N_T$ which grows exponentially with the number of vehicles. For example, in a scenario with four lanes and two vehicles, the number of homotopy candidates is 54. However, some of these candidates are not feasible due to violation of constraints associated with the vehicle dynamics, collisions, or road and lane boundaries. Furthermore, homotopy candidates associated with multiple lane changes can be discarded as practically undesirable.

The identification of these unfeasible homotopy candidates can be performed using the threat assessment methods described in the next section that relies on sets of trajectories to compute the control margin within each corridor. However, this computation of the margin can become computationally expensive as the number of candidates increases quickly. Instead, it is possible to prune the number of candidates before performing the threat assessment task. A fraction of the candidates can quickly be sorted out based on heuristics such as the minimum number of lane changes associated with the maneuver.

As an example, consider the scenario illustrated in Fig. 3., with two obstacles and two identified candidate homotopies. We can see that bypassing vehicle A (from left to right) requires passing through lane 1 or 2. Approaching behind vehicle A requires travel in lane 3 and overtaking it from the right side requires travel in lane 4. Based on these observations and from a conservative perspective, we can

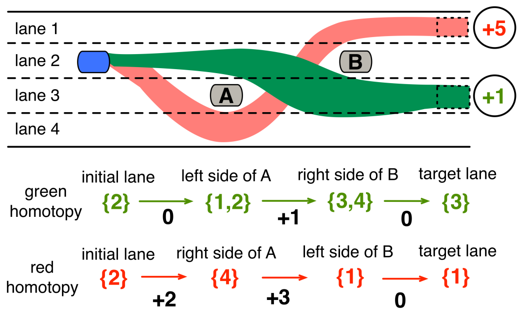

Fig. 3. examples of computing minimum number of lane changes for homotopy candidates

then calculate the minimal number of lane changes involved in the maneuver. The red homotopy requires at least 5 lane changes, whereas the green homotopy minimally requires only one. We can thus discard the red option from the homotopy candidates as it is unlikely that a) it contains any feasible paths, considering constraints arising from vehicle dynamics, and b) the large number of required lane changes makes it practically unlikely to be desirable to a driver.

Let L_j be the set of accessible lanes corresponding to a decision for the j^{th} vehicle. For example, when the j^{th} vehicle is in lane 3 and decision is *'left'*, then L_j is $\{1,2\}$. For notational convenience, let L_0 be the initial lane and let L_{Nv+1} be the target lane. The minimum number of lane changes between decisions can be computed as the minimum value of differences between elements of L_j and L_{j+1}, which we denote $min\_diff(L_j, L_{j+1})$. Then the total minimum number of lane changes can be computed by summing them, as:

$$Min = \sum_0^{Nv} min\_diff(L_j, L_{j+1})$$

III. THREAT ASSESSMENT PROBLEM

The second stage of the threat assessment problem is to characterize the desirability of each candidate homotopy that was previously identified. The threat assessment approach described here relies on a metric that is based on the volume of the feasible input space bounded by problem constraints, such as spatial boundaries or vehicle dynamic constraints. A lattice structure sampled in input space is implemented to characterize the input space. Lattice structures are adopted since they can flexibly consider complex scenarios with multiple inputs (i.e. change of heading and velocity).

A. Lattice Structure

Lattice structures are used to enable the use of discrete graph search techniques on a continuous state or input space representation, while respecting differential constraints on motion [7]. Most previous work presents conformal state lattices whose vertices are placed in a regular pattern and connected by cubic polynomial paths. In [12] and [15], the sets of trajectories are generated in two steps. First, the algorithm samples endpoints along the road and computes the connections, generating physical paths along which the vehicle can travel at any desired speed. Then, for each path, a set of trajectories is defined by specifying different acceleration profiles. A trajectory therefore contains information about the coordinates of the path (spatial information) and the velocity at which the vehicle travels the path (temporal information). State lattices may require re-computing the entire lattice at each time step, unlike lattices derived from input-space sampling, which do not depend on the environment constraints.

Here we adopt a discretization of the input space, since this approach is easy to implement and allows easy storage of pre-computed lattices, avoiding the computationally expensive construction requirement at each time step. If the road contains important curvature changes, the entire lattice can be shifted to fit the road shape more precisely.

1) Vehicle Model: The vehicle model used to build the lattice accounts for the kinematics of a 4-wheeled Ackermann-steered vehicle, along with its lateral and yaw dynamics. It is the same model employed in [3]; the reader may refer to this paper for a description of the equations of motion. The two inputs to control the vehicle are the forward speed (throttle) and steering angle. Tire compliance is included in the model by approximating the lateral tire force as the product of wheel cornering stiffness and wheel sideslip. Note that the equations of motion are linearized about a constant speed and assume small slip angles. The global velocity profile is thus a piecewise linear function.

2) Construction of the lattice: The resolution of the lattice discretization is chosen so that the change in input at each vertex is compatible with both vehicle dynamic constraints (i.e. available acceleration or deceleration level) and steering constraints (steering rate bounds). Each level of the lattice corresponds to a specific time step of a discretized, user-selected time horizon. At each time step, the planner may switch input in both steering and speed. The main limitation of lattice structures is the curse of dimensionality: the complexity of the tree grows exponentially with the resolution and thus the number of levels in the tree. If N_δ and N_V are, respectively, the number of discretized steering inputs and available speed levels at each node, and L the number of levels in the tree, then the total number of trajectories stored in the lattice is $\Sigma = (N_\delta \times N_V)^L$.

In order to build a lattice that is sufficiently dense to cover the space defined by the homotopy candidates, while not exceeding a reasonable computational threshold in terms of number of trajectories, we take into account the following remarks:

- The initial time steps in the sequence of inputs have more influence than the final ones on the trajectory's shape.
- The bounds of the velocity rate are typically greater for deceleration than for acceleration.

For these two reasons, we reduce the resolution of the input space from the upper levels of the lattice to the lower levels: here, seven steering inputs are defined at the first level compared to three at the final level. Also, at each node, we define four available speed inputs: $+0m/s$, $+2m/s$, $-2m/s$ and $-4m/s$. We define a four-level lattice, which enables a range of typical maneuvers, including double lane changes. The horizon time is chosen as three seconds. Overall, the total number of trajectories stored in the lattice amounts to 10^5, which results in an algorithm that is practically computationally feasible and yields a lattice that is sufficiently dense to cover the vehicle's local environment.

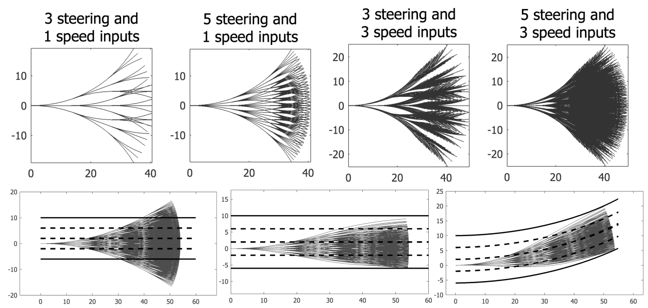

Fig. 4. Top: Construction of the three-level lattice for various resolutions. Bottom: Shifting the lattice to fit the road shape

3) Fitting the shape of the road: Contrary to state-space lattices that can be conformed to the road geometry by construction, input-space lattices can potentially generate trajectories whose endpoints lie outside the road boundaries or whose final yaw angles deviate significantly from the mean road heading angle. To address this issue, the lattice is subjected to a transformation that is based on properties of the road centerline. An algorithm is employed to check (in a brute force manner) that all trajectories remain consistent with input constraints and stability vehicle bounds. This allows use of a lattice that conforms to the road geometry while maintaining a relatively low computation cost. Fig. 4 illustrates the process of pruning inconsistent trajectories and conforming the lattice to the road.

B. Estimation of the Margin of control

The proposed margin-based approach to threat assessment relies on the assumption that threat to a vehicle is inversely proportional to the available control freedom, and therefore to the maneuverability within a candidate homotopy. For each candidate homotopy, the sequence of decisions from the decision tree can be directly mapped to inequality constraints in the vehicle's input space. These inequality constraints yield 2-dimensional convex polytopes bounding the range of feasible vehicle inputs. Essentially, if we can compute the volume inside this constrained region, we expect that this will result in a rigorous metric to characterize the available control freedom.

A planner searches the lattice to identify trajectories that feasibly conform to each unique homotopy candidate. In some cases, a homotopy candidate contains no feasible trajectories and thus the associated navigation decision is said to be infeasible. We note N_k is the number of feasible trajectories in the k^{th} corridor. The associated set is called ψ_k and each trajectory from the set is referred as $\psi_{k,i}$ with i ranging from 1 to N_k. These trajectories take on discrete values until the finite horizon time T. Instead of considering each trajectory independently, we consider $\psi_{k,1:N_k}(t)$ for each t. Then, at each time step, the associated values are plotted in the vehicle's acceleration space, creating a scatter plot. Computing the volume in acceleration space has the advantage of being homogenous, compare to a typical input space with the speed on the one hand and the steering command on the other hand. To compute the margin of control at this time step and for a specific corridor, one can simply compute the convex hull of the scatter plot. A useful metric can then be defined as the radius of the Chebyshev ball that lies within the convex hull.

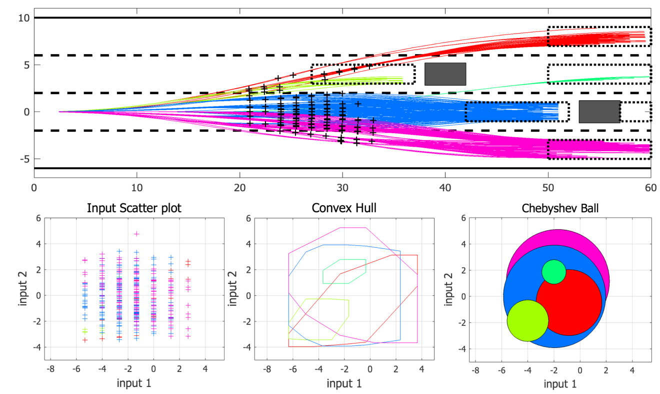

Fig. 5. Top: sorting of the trajectories according to homotopy belonging. Bottom: Computation of the convex hulls and Chebyshev balls

Fig. 5. illustrates how distinct trajectories from a lattice are sorted according to their membership in a particular homotopy. In this case, among all homotopy candidates, only five are feasible. On the top figure, black dots show the potential locations of the host vehicle at a specific time step (t=1.5 s). The black scatter plot is then mapped into the vehicle's acceleration space. The first step consists of computing the convex hull of each group of points, whose area is one possible metric to estimate the margin of the homotopy. Another plausible metric is the area of the Chebyshev ball, i.e. the largest ball circumscribed in the convex hull. The center figure shows the convex hull and the right figure the Chebyshev balls. The two metrics are similar in principle but can yield different results depending on the region geometric properties. For example, an area metric can be interpreted as assessing the average margin of the two acceleration inputs, whereas the Chebyshev metric can be interpreted as assessing the minimal margin of the two acceleration inputs. (Thus, a long rectangular space with a small width can have a large area while the circumscribed ball has a small radius.) Depending on the modeling of how drivers perceive threat, one or the other can be used.

Each maneuver is defined over a given time horizon. However, the final goal is to assess the overall margin, and not its value at a specific instant. The previous mechanism is therefore repeated at every time step. Various norms can then be used to normalize the margin, such as the average, Euclidian or minimum norm. Fig. 6. shows the Chebyshev balls associated with the homotopy candidates at different

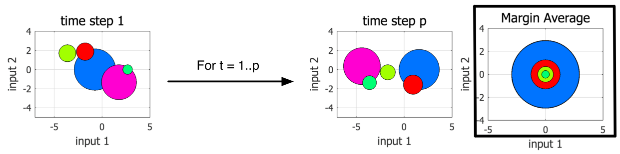

Fig. 6. Computation of the overall margin

time step. The overall margin is a normalization of the individual computations.

IV. Simulation Testing in Highway Scenarios

The proposed threat assessment method was studied in a Matlab simulation environment using the 4-wheeled Ackermann-steered model to simulate a standard passenger vehicle, whose weight is 1100kg and dimensions are 4 by 2 meters. The maximal steering angle was limited to 10 deg and the lateral acceleration to 0.7g. The prediction horizon was set to 3 seconds and discretized into 20 time steps (i.e. sampling time of 150 ms). The lattice was chosen as a four level-tree, meaning each edge of the lattice spans five time steps. The reference speed was set to 20 m/s. The number of lanes, the curvature of the road as well as the position and speed of other vehicles were left customizable.

A. Static obstacles

The first example is the same as the one introduced in the previous section, i.e. travel on a four lane road with two static obstacles present in the second and third lanes. The vehicle of interest is originally located in the third lane. In Fig. 7., the center of each disc corresponds to the possible location of the vehicle in the corridor at a specific time step, and the radius is proportional to the computed freedom of control. For simplicity sake, only one representative of each set of trajectories is plotted. This is the feasible path from the set corresponds to the location at the Chebyshev center of the convex hull. Although the value of the metric assessing the margin of control is time varying, the area of the discs of the same homotopy are here shown as a constant (computed as the average over the entire horizon) to improve the clarity of presentation. The right figure shows the temporal variation of the control margin in each homotopy candidate.

Clearly, the blue and purple homotopies represent fields of travel with the greatest maneuver freedom. As expected, the dark-green strategy is the most hazardous as it requires swerving between two obstacles that are near each other. The two other choices appear less desirable due to a stronger restriction of the input space: strong deceleration to stop behind the obstacle for the light green homotopy, and a severe steering maneuver for the red homotopy (to bypass the top-most obstacle on the left).

Fig. 8. displays the computed margin considering either the area of the convex hull (top) or the radius of the Chebyshev ball (bottom) for various norms. In this scenario, all metrics yield an identical hierarchy regarding the degree of control freedom afforded by each homotopy candidate. The average norm and Euclidian norm yield similar results, while

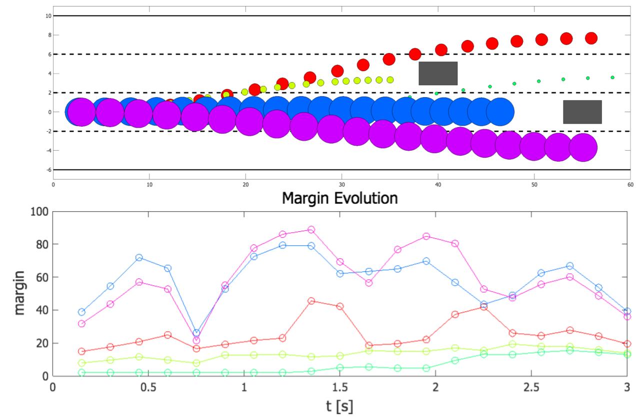

Fig. 7. Display and evolution of the margin

minimum norm tends to minimize the difference between the computed margins. The Chebyshev metric tends to increase the difference between the two most desirable homotopy candidates and the next two, compared to the convex hull metric. The reason is that it depends on the minimal margin in both input dimensions, whereas the convex hull metric relies on the global volume, independent of the distribution. In other words, homotopy candidates four and five have a large margin in both inputs whereas candidates one and two have a relatively comfortable margin only in one direction (either steering for candidate 2 or throttle for candidate 1).

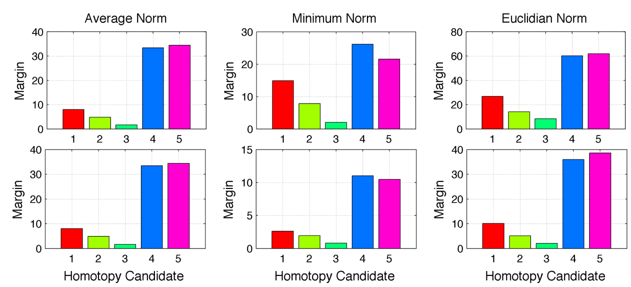

Fig. 8. Norm comparison and hierarchy of the desireness of homotopy candidates, top = convex hull, bottom = Chebyshev ball

B. Evolution of threat

The first example considers the control margin in each candidate homotopy at a particular instant of the driving task. A threat estimate based on margin can then be leveraged to plan the most desirable navigation option for an autonomous system, or as a means to modulate a human's input in a semi-autonomous driver assistance system. To analyze how threat assessment in each corridor evolves with time, we extend the previous example by assuming that the operator simply travels straight at a constant speed. As a result, the threat posed to the vehicle increases as it nears the obstacles without reacting. The figure below displays the homotopy candidates and the associated control margin at three different time steps, corresponding to distances of 25m, 40m, 55m to the obstacle in lane 3.

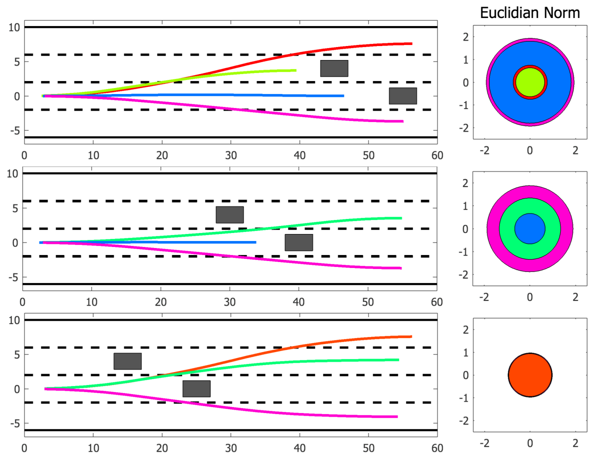

Fig. 9. Evolution of the threat

In the absence of a nearby obstacle, the most desirable navigation decision is to drive straight at a constant speed. As the vehicle nears the obstacle, changing lanes becomes the most desirable navigation decision, as indicated by the larger margin for avoidance maneuvers shown in the middle subfigure. There, remaining in the lane requires a strong braking input and loses its attractiveness compared to the (purple and green) candidate homotopies that involve lane-change maneuvers. (Note that the green corridor exhibits reduced margin due to the presence of a second obstacle on the second lane.) Finally, when the vehicle reaches a distance of 25m to the obstacle, it has no option but to change lanes because there is insufficient braking authority to stop the vehicle and avoid collision.

C. Dynamic Environment

The final example involves a more complex scenario that includes dynamic obstacles, lane merging and a lane-shifting vehicle. In this scenario the fourth lane is being eliminated, forcing the vehicle in the extreme right to merge to the third lane, which is the current lane of the host vehicle. As in the static case, the homotopy candidates are displayed along with the associated control margin. We here assume perfect knowledge of the state evolution of the dynamic obstacles. In the dark blue case, the host vehicle travels straight and accelerates to pass ahead of the vehicle in the fourth lane, before it merges into the third lane. In the light blue navigation strategy, the host vehicle brakes to allow the other vehicle to merge in front. The green homotopy candidate offers the largest margin and corresponds to the case where the vehicle merges to the second lane to avoid the merging vehicle entirely. The red and yellow candidates offer comparatively little control freedom.

V. CONCLUSION

A margin-based approach for threat assessment has been introduced that yields reasonable results and can accommodate complex scenarios with dynamic environments. A lattice structure is employed that provides a flexible manner to characterize the control freedom associated with candidate homotopies. Future work will focus on methods for integrating this threat estimate information into semi-autonomous or autonomous control architectures.

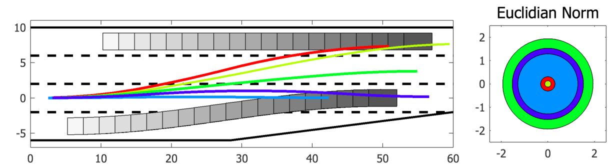

Fig. 10. Homotopy candidates and associated control freedom

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